1. (a) $A^*A = (I - 2vv^*)^* (I - 2vv^*)$

$= (I - 2vv^*)(I - 2vv^*)$

$= I - 4vv^* + 4vv^*vv^*$

since $\|v\|_2 = 1$, i.e. $v^*v = \|v\|_2 = 1$

$\Rightarrow A^*A = I - 4vv^* + 4v(v^*v)v^*$

$= I - 4vv^* + 4vv^*$

$= I$

$\Rightarrow$ A is unitary.

(b) $Av = (I - 2vv^*)v$

$= v - 2v(v^*v)$

$= v - 2v$

$= -v$

$\Rightarrow$ A has an eigenvalue to -1.

(c) Consider $\{v\}^\perp$. of dimention $n-1$

$\forall u \in \{v\}^\perp \Rightarrow v^*u = 0$

$Au = (I - 2vv^*)u$

$= u - 2v(v^*u)$

$= u$

$\Rightarrow 1$ is A's eigen value

and the geometric multiplicity is at least $n-1$

Since -1 is also an eigenvalue and has geometric multiplicity at least 1

and the sum of two geometric multiplicity $\leq n$

$\Rightarrow$ The geometric multiplicity of 1 is $n-1$

and the algebric multiplicity is also $n-1$.

and the reason is same as above

(algebric $\geq$ geometric multiplicity)