

$$\begin{aligned}
 1. \quad (a) \quad A^*A &= (I - 2vv^*)^* (I - 2vv^*) \\
 &= (I - 2vv^*) (I - 2vv^*) \\
 &= I - 4vv^* + 4vv^*vv^*
 \end{aligned}$$

since $\|v\|_2 = 1$, i.e. $v^*v = \|v\|_2^2 = 1$

$$\begin{aligned}
 \Rightarrow A^*A &= I - 4vv^* + 4v(v^*v)v^* \\
 &= I - 4vv^* + 4vv^* \\
 &= I
 \end{aligned}$$

$\Rightarrow A$ is unitary.

$$\begin{aligned}
 (b) \quad Av &= (I - 2vv^*)v \\
 &= v - 2v(v^*v) \\
 &= v - 2v \\
 &= -v
 \end{aligned}$$

$\Rightarrow A$ has an eigenvalue -1 .

(c) Consider $\{v\}^\perp$ of dimension $n-1$

$$\forall u \in \{v\}^\perp \Rightarrow v^*u = 0$$

$$\begin{aligned}
 Au &= (I - 2vv^*)u \\
 &= u - 2v(v^*u) \\
 &= u
 \end{aligned}$$

$\Rightarrow 1$ is A 's eigen value

and the geometric multiplicity is at least $n-1$

Since -1 is also an eigenvalue and has

geometric multiplicity at least 1
and the sum of two geometric multiplicities $\leq n$
 \Rightarrow The geometric multiplicity of 1 is $n-1$

and the algebraic multiplicity is also $n-1$.

and the reason is same as above

(algebraic \geq geometric multiplicity)