

Math 2411: Final examination (Spring 2021)

- This exam is **2 hours long**.
- Your numerical answers should be EITHER exact OR correct to 3 decimal places.
- For each question, you must show all of your working steps in order to receive full points. Zero point will be given if you only provide an answer without any working steps.
- **Cheating is a serious offense**. Students caught cheating are subject to a zero score as well as additional penalties.

The HKUST Academic Honor Code

Honesty and integrity are central to the academic work of HKUST. Students of the University must observe and uphold the highest standards of academic integrity and honesty in all the work they do throughout their program of study.

As members of the University community, students have the responsibility to help maintain the academic reputation of HKUST in its academic endeavors.

Sanctions will be imposed on students, if they are found to have violated the regulations governing academic integrity and honesty.

Declaration of Academic Integrity

I confirm that I have answered the questions using only materials specifically approved for use in this examination, that all the answers are my own work, and that I have not received any assistance during the examination.

Student's Signature:

You can also handwrite/type the above RED statements, and then sign and upload your file together with the file(s) of your answers and R codes to canvas.

Scores						
Q1	Q2	Q3	Q4	Q5	Q6	Total
/12	/15	/15	/12	/15	/15	/84

Question 1

[12 pts] Use MME with the following data

1, 1, 0, 0, 0, 2, 1, 2, 2, 1, 0, 1

to estimate the parameters θ_1 and θ_2 for the distribution of X whose moment generating function is given by

$$M_X(t) = (1 - \theta_1 - \theta_2) + \theta_1 e^t + \theta_2 e^{2t}.$$

Question 2

A cake maker produces special mint cakes. Assume that the distribution of the weights of these cakes is normal with mean 21.5 grams and standard deviation 0.4 gram.

- [2 pts] Find the probability that the weight of a randomly selected mint cake is fewer than 21 grams.
- [3 pts] Suppose that 8 mint cakes are selected randomly and weighed independently. Find the probability that exactly three of these cakes will weigh fewer than 21 grams.
- [3 pts] Suppose that 37 mint cakes weighed independently and are packed into a bag for sale. Find the probability that a randomly selected bag will contain more than 11% of the cakes weighing fewer than 21 grams.
- [7 pts] Suppose that 24 mint cakes are selected randomly and weighed independently. Find the probability that their total weight will be greater than 520 grams.

Question 3

Use less than 30 words to answer each of the following questions:

- [3 pts] What are data?
- [3 pts] Why do we need to collect data?
- [3 pts] What do we want to study when a regression model is used?
- [3 pts] What do we want to know when an ANOVA model is used?
- [3 pts] Comment \bar{X} and \bar{x} .

Question 4

Read the data from the file exam_data.txt.

We study the trend of daily-confirmed cases with time (time index).

- [2 pts] Write down a fitted regression line.
- [1 pt] What is the R^2 of the fitted line?
- [2 pts] Construct 99% C.I.s for β_0 and β_1 .
- [2 pts] Test $H_0: \beta_0 = 0$ against $H_1: \beta_0 \neq 0$ at 0.01 level of significance. Please draw your conclusion with evidence.
- [3 pts] Test $H_0: \beta_1 = 1$ against $H_1: \beta_1 > 1$ at 0.01 level of significance. Please draw your conclusion with evidence.
- [2 pts] Predict the number of confirmed cases for Jul 18 and find its residual.

Question 5


Consider

X1	X2	X3
X4	X5	X6
X7	X8	X9

, where X1 – X9 are the features of an image.

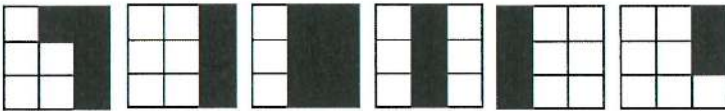
Only two colours (black and white) are used for each square in the image. 1 is then assigned to the feature if black is used, and 0 is assigned for white.



For instance, for the image , we have $(X1, X2, X3, X4, X5, X6, X7, X8, X9) = (0, 1, 1, 0, 0, 1, 0, 0, 1)$.

Suppose that the images are labelled as either "1" or "7".

Images labelled as "1"



Images labelled as "7"



Use the above images as training data with Naïve Bayes Classifier to find the predictive probabilities for "1" and "7", and then determine the predictive number (or label) of the following image



Answer this question

- a) [5 pts] with R
- b) [10 pts] without R

Question 6

Read the data from the file exam_data.txt.

- a) [3 pts] Find the sample mean of the daily-confirmed cases for each week.
- b) Suppose the daily-confirmed cases for each week follow normal distributions with equal variances. They are also assumed to be independent.

[3 pts] Test whether the means of the daily-confirmed cases in different weeks are all equal or not at a significant level of 0.03.

- c) Suppose the assumption of normal distributions cannot be made. Test whether the medians of the daily-confirmed cases in different weeks are all equal or not at a significant level of 0.03.
 - (i) [3 pts] Find the value of F_r .
 - (ii) [3 pts] Use the result in (i) to find the value of the test statistic H_r .
 - (iii) [3 pts] Draw your conclusion with evidence.